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roots of (2) are easily found to be $x=1, x=2, x=3, x=4$. We have, therefore, the following four groups of values:

$$\begin{cases} x=1, & y=2, & z=3, & u=4; \\ x=2, & y=1, & z=4, & u=3; \\ x=3, & y=4, & z=1, & u=2; \\ x=4, & y=3, & z=2, & u=1. \end{cases}$$

Also solved by G. B. M. Zerr, J. Scheffer, and A. H. Holmes.

GEOMETRY.

308. Proposed by W. J. GREENSTREET, M. A., Editor of *The Mathematical Gazette*, Stroud, England.

Find the locus of O , if the differences of the squares of tangents from it to circles A, B, C are x^2, y^2, z^2 , respectively.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.; A. H. HOLMES, Brunswick, Maine; L. E. NEWCOMB, Los Gatos, California; and J. SCHEFFER, A. M., Hagerstown, Md.

Let (u, v) be the co-ordinates of the point O ;

$$(x-m_1)^2 + (y-n_1)^2 = R_1^2, \text{ the equation of the circle } A,$$

$$(x-m_2)^2 + (y-n_2)^2 = R_2^2, \text{ the equation of the circle } B, \text{ and}$$

$$(x-m_3)^2 + (y-n_3)^2 = R_3^2, \text{ the equation of the circle } C.$$

Then $(u-m_1)^2 + (v-n_1)^2 - R_1^2 = T_1^2$, the square of the tangent from O to A ;

$$(u-m_2)^2 + (v-n_2)^2 - R_2^2 = T_2^2, \text{ the square of the tangent from } O \text{ to } B;$$

$$(u-m_3)^2 + (v-n_3)^2 - R_3^2 = T_3^2, \text{ the square of the tangent from } O \text{ to } C.$$

$$\therefore 2(m_2 - m_1)u + 2(n_2 - n_1)v + m_1^2 + n_1^2 - m_2^2 - n_2^2 + R_2^2 - R_1^2 = x^2 \dots (1),$$

$$2(m_3 - m_1)u + 2(n_3 - n_1)v + m_1^2 + n_1^2 - m_3^2 - n_3^2 + R_3^2 - R_1^2 = y^2 \dots (2),$$

$$2(m_3 - m_2)u + 2(n_3 - n_2)v + m_2^2 + n_2^2 - m_3^2 - n_3^2 + R_3^2 - R_2^2 = z^2 \dots (3).$$

Adding (1) and (3) and subtracting (2), we have $x^2 + z^2 = y^2$ or $y^2 - x^2 = z^2$, the former being the equation of a circle with a variable radius y , and the latter the equation of an equilateral hyperbola with a variable semi-axes z .

If $x, y,$ and z are constants, the locus is a point.

309. Proposed by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

To find the equation of Brocard's Ellipse, the sides b and c of the triangle being the axes of co-ordinates.

Solution by the PROPOSER.

Let AB be the axis of x, AC that of y , then the equation of any ellipse touching the three sides of the triangle is of the form $D^2y^2 + 4Bxy + E^2x^2 + 4Dy + 4Ex + 4 = 0$, where

$$B = -\frac{8 + 4bD + 4cE + bc DE}{2bc}.$$

The co-ordinates of the center of the ellipse are $-\frac{2D}{DE+2B}$ and $-\frac{2E}{DE+2B}$, or substituting the value of B , $\frac{Dbc}{2(2+bD+cE)}$ and $\frac{Ebc}{2(2+bD+cE)}$. The co-ordinates of the two Brocard Points O and O' are, respectively, $\frac{a^2b^2c}{a^2b^2+a^2c^2+b^2c^2}$, $\frac{b^3c}{a^2b^2+a^2c^2+b^2c^2}$, and $\frac{b^2c^3}{a^2b^2+a^2c^2+b^2c^2}$, $\frac{a^2c^2b}{a^2b^2+a^2c^2+b^2c^2}$.

Therefore, the co-ordinates of the middle point of OO' , that is, the center of the ellipse, are

$$\frac{1}{2} \cdot \frac{b^2c(a^2+c^2)}{a^2b^2+a^2c^2+b^2c^2} \quad \text{and} \quad \frac{1}{2} \cdot \frac{bc^2(a^2+b^2)}{a^2b^2+a^2c^2+b^2c^2};$$

$$\therefore \frac{D}{2+bD+cE} = \frac{b(a^2+c^2)}{a^2b^2+a^2c^2+b^2c^2}, \quad \frac{E}{2+bD+cE} = \frac{c(a^2+b^2)}{a^2b^2+a^2c^2+b^2c^2};$$

whence, $D = -\frac{2(a^2+c^2)}{bc^2}$, $E = -\frac{2(a^2+b^2)}{b^2c}$.

Substituting, we find for the required equation of the Brocard Ellipse,

$$b^2(a^2+c^2)^2y^2 + 2bc[(a^2b^2+a^2c^2+b^2c^2)-a^4]xy + c^2(a^2+b^2)^2x^2 - 2b^3c^2(a^2+c^2)y - 2b^2c^3(a^2+b^2)x + b^4c^4 = 0.$$

Also solved by G. B. M. Zerr, who used trilinear co-ordinates.

310. Proposed by L. H. MacDONALD, A. M., Ph. D., Sometime Tutor in the University of Cambridge, Jersey City, N. J.

Construct a plane triangle having given the base, the vertical angle, and the bisector of the vertical angle.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.; J. SCHEFFER, A. M., Hagerstown, Md., and C. N. SCHMALL, A. B., 89 Columbia Street, New York.

Upon the given base AB construct a circle whose segment ACB shall contain the given vertical angle. Through E , the mid-point of AB , draw EF perpendicular to AB , meeting the circumference at F . Join FB , and perpendicular to FB draw BG equal to one half the given bisector of the vertical angle. With G as center and BG as radius describe the circle BHL , and draw FGL . With F as center, FL as radius, describe a circle cutting the given circle in C . Join FC , cutting AB in D . Then ABC is the triangle required.

