roots of (2) are easily found to be $x=1$, $x=2$, $x=3$, $x=4$. We have, therefore, the following four groups of values:

$$\begin{cases} x=1, & y=2, & z=3, & u=4; \\ x=2, & y=1, & z=4, & u=3; \\ x=3, & y=4, & z=1, & u=2; \\ x=4, & y=3, & z=2, & u=1. \end{cases}$$

Also solved by G. B. M. Zerr, J. Scheffer, and A. H. Holmes.

GEOMETRY.

308. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Find the locus of O, if the differences of the squares of tangents from it to circles A, B, C are x^2, y^2, z^2, respectively.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.; A. H. HOLMES, Brunswick, Maine; L. E. NEWCOMB, Los Gatos, California; and J. SCHEFFER, A. M., Hagerstown, Md.

Let (u, v) be the co-ordinates of the point O;

$(x-m_1)^2+(y-n_1)^2=R_1^2$, the equation of the circle A,

$(x-m_2)^2+(y-n_2)^2=R_2^2$, the equation of the circle B, and

$(x-m_3)^2+(y-n_3)^2=R_3^2$, the equation of the circle C.

Then $(u-m_1)^2+(v-n_1)^2-R_1^2=T_1^2$, the square of the tangent from O to A;

$(u-m_2)^2+(v-n_2)^2-R_2^2=T_2^2$, the square of the tangent from O to B;

$(u-m_3)^2+(x-n_3)^2-R_3^2=T_3^2$, the square of the tangent from O to C.

$\therefore 2(m_2-m_1)u+2(n_2-n_1)v+m_1^2+n_1^2-m_2^2-n_2^2+R_2^2-R_1^2=x^2 \ldots (1)$,

$2(m_3-m_1)u+2(n_3-n_1)v+m_1^2+n_1^2-m_3^2-n_3^2+R_3^2-R_1^2=y^2 \ldots (2)$,

$2(m_3-m_2)u+2(n_3-n_2)v+m_2^2+n_2^2-m_3^2-n_3^2+R_3^2-R_2^2=z^2 \ldots (3)$.

Adding (1) and (3) and subtracting (2), we have $x^2+z^2=y^2$ or $y^2-x^2=z^2$, the former being the equation of a circle with a variable radius y, and the latter the equation of an equilateral hyperbola with a variable semi-axes z.

If x, y, and z are constants, the locus is a point.

309. Proposed by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

To find the equation of Brocard's Ellipse, the sides b and c of the triangle being the axes of co-ordinates.

Solution by the PROPOSER.

Let AB be the axis of x, AC that of y, then the equation of any ellipse touching the three sides of the triangle is of the form $D^2y^2+4Bxy+E^2x^2+4Dy+4Ex+4=0$, where

$$B=-\frac{8+4bD+4cE+bcDE}{2bc}.$$

The co-ordinates of the center of the ellipse are $-\frac{2D}{DE+2B}$ and $-\frac{2E}{DE+2B}$, or substituting the value of B, $\frac{Dbc}{2(2+bD+cE)}$ and $\frac{Ebc}{2(2+bD+cE)}$. The co-ordinates of the two Brocard Points O and O' are, respectively,

$$\frac{a^2b^2c}{a^2b^2+a^2c^2+b^2c^2},\ \frac{b^3c}{a^2b^2+a^2c^2+b^2c^2},\ \text{and}\ \frac{b^2c^3}{a^2b^2+a^2c^2+b^2c^2},\ \frac{a^2c^2b}{a^2b^2+a^2c^2+b^2c^2}.$$

Therefore, the co-ordinates of the middle point of OO', that is, the center of the ellipse, are

$$\tfrac{1}{2}.\frac{b^2c(a^2+c^2)}{a^2b^2+a^2c^2+b^2c^2}\ \text{and}\ \tfrac{1}{2}.\frac{bc^2(a^2+b^2)}{a^2b^2+a^2c^2+b^2c^2};$$

$$\therefore\ \frac{D}{2+bD+cE}=\frac{b(a^2+c^2)}{a^2b^2+a^2c^2+b^2c^2},\quad \frac{E}{2+bD+cE}=\frac{c(a^2+b^2)}{a^2b^2+a^2c^2+b^2c^2};$$

whence, $D=-\frac{2(a^2+c^2)}{bc^2}$, $E=-\frac{2(a^2+b^2)}{b^2c}$.

Substituting, we find for the required equation of the Brocard Ellipse,

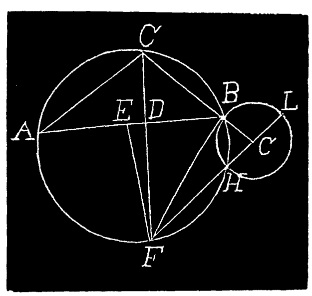
$$b^2(a^2+c^2)^2y^2+2bc[(a^2b^2+a^2c^2+b^2c^2)-a^4]xy+c^2(a^2+b^2)^2x^2$$
$$-2b^3c^2(a^2+c^2)y-2b^2c^3(a^2+b^2)x+b^4c^4=0.$$

Also solved by G. B. M. Zerr, who used trilinear co-ordinates.

310. Proposed by L. H. MacDONALD, A. M., Ph. D., Sometime Tutor in the University of Cambridge, Jersey City, N. J.

Construct a plane triangle having given the base, the vertical angle, and the bisector of the vertical angle.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.; J. SCHEFFER, A. M., Hagerstown, Md., and C. N. SCHMALL, A. B., 89 Columbia Street, New York.



Upon the given base AB construct a circle whose segment ACB shall contain the given vertical angle. Through E, the mid-point of AB, draw EF perpendicular to AB, meeting the circumference at F. Join FB, and perpendicular to FB draw BG equal to one half the given bisector of the vertical angle. With G as center and BG as radius describe the circle BHL, and draw FGL. With F as center, FL as radius, describe a circle cutting the given circle in C. Join FC, cutting AB in D. Then ABC is the triangle required.