The Origin and Growth of Ripple-mark.

By Mrs. Hertha Ayrton.

(Communicated by the late Prof. W. E. Ayrton, F.R.S. Received April 21, 1904; received in revised form May 26, 1904; read June 16, 1904.*)

[Plates 3 and 4.]

To any one who, for the first time, sees a great stretch of sandy shore covered with innumerable ridges and furrows, as if combed with a giant comb, a dozen questions must immediately present themselves. How do these ripples form? Are they made and wiped out with every tide, or do they take a long time to grow, and last for many tides? What is the relation between the ripples and the waves to which they owe their existence? And a host of others too numerous to mention.

The questions to which I particularly directed my attention at first were the following:—(1) How do the ripples first start? (2) What is the relation between the water waves and the ripples?

During the course of this investigation certain fresh facts have come to light, showing how the principles involved in the formation of ripple-mark apply to other phenomena of apparently widely different origin. Some of these are included in the present communication, but the discussion of others, less immediately connected with ripples, I have deferred to a future occasion.

1. *Starting of the First Ripple.*—To the first question as to the *origin* of ripple-mark—fundamental as it is—I could, for some time, find no satisfactory answer, either in nature or in books. Even the deeply interesting paper† in which Prof. George Darwin described the vortices he had discovered in the water oscillating over ripple-mark touched but lightly on this point. Prof. Darwin said: "When a small quantity of sand is sprinkled [in a glass trough] and the rocking begins, the sand dances backwards and forwards on the bottom, the grains rolling as they go.

"Very shortly the sand begins to aggregate into irregular little flocculent masses, the appearance being something like that of curdling milk. *The position of the masses is, I believe, solely determined by the friction of the sand on the bottom.*"

And again: "We now revert to the initiation of ripple-mark.

"If the surface be very even, as when sand is sprinkled on glass, when a uniform oscillation of considerable amplitude be established, the sand is

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* [Publication postponed by author's desire until June, 1910.]

† "On the Formation of Ripple-mark in Sand," 'Roy. Soc. Proc.,' November 22, 1883, vol. 36, p. 23.

carried backwards and forwards and *some of the particles stick in places of greater friction. As soon as there is any superficial inequality,* it is probable that a vortex is set up in the lee of the inequality which tends to establish a dune there." [The italics are mine.]

With all the respect I felt for so eminent an observer—one, also, who had thrown so much light on the subject—I could not concur in this opinion. It seemed to me impossible that chance inequalities, having no relation with one another, but scattered here and there entirely without order, should develop into such ripples as are commonly seen on the sea shore—straight as if ruled, all of the same shape, and all at equal distances apart, or at distances varying according to some definite law. I cast about, therefore, for some other solution to the problem—some way of connecting the ripples with one another from the beginning, without the intervention of chance irregularities in the sand. I may say at once that I have been successful in finding such a solution, and that I am about to show how oscillating water can produce ripple-mark on sand which is perfectly smooth and level to start with, and free from irregularity of every sort.

My first experiments were carried out at Margate, with the rather coarse brown sand found there. I tried oscillating water of various depths, over different thicknesses of sand, in vessels of all sorts of shapes and sizes, from a soap dish some 4″ × 3″ × 2″ to a tank 44″ × 18″ × 18″. The oscillations were produced by giving the vessel either slight instantaneous horizontal pushes, or a very small rocking motion, in time with the natural swing of the water,* and by putting either rollers or cushions under the vessel to ease the jerks. The sand was made quite level at the beginning of each experiment, by being violently stirred up first, and then gently and irregularly shaken while it was settling.

In every vessel ripples appeared in times varying from a few seconds to a few minutes; and in all those in which the water was simply made to rise and fall alternately at each end of the vessel, without the formation of intermediate waves, two things invariably happened:—(1) Ripples formed first across the *middle* of the vessel, a fact first observed by C. de Candolle,† and (2) after prolonged oscillation most of the sand had collected there also in a long ripple-marked heap, as shown in fig. 1 (Plate 3).

Since ripples formed first across the middle of the vessel, and as, also, the sand was gradually removed from near the ends to the middle, by prolonged

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* When each impulse is prolonged, even although the impulses be in time with the natural vibration of the water, the effect on the sand is quite different, for a reason that will be given in another paper.

† "Rides Formées à la Surface du Sable Déposé au Fond de l'Eau," 'Archives des Sciences Physiques et Naturelles,' Période 3, vol. 9, p. 253 (1883).

oscillation, it seemed clear that it was the formation of a small ridge* across the middle, during the first few oscillations, that caused the ripples to start there first.

In order both to reduce surface friction, and to render observation easy, I repeated the experiment, scattering a mere pinch of sand as evenly as possible over the smooth bottom of a pie dish, some 8 inches long, containing about an inch of water. In this way, each grain, being isolated from the next, could be easily moved by the water, and as readily observed. The result was very striking. After oscillation of the water for less than half a minute, *the whole of the sand was collected in a straight line across the middle of the dish at right angles to the line of motion.*

Scattering the sand again, and watching carefully how the water moved it, I saw that each swing† of the water pushed every grain that was being swept *towards* the middle farther than the next swing carried it *away* again from the middle. If *a*, for instance (fig. 2), were the position of a grain at the beginning, and *b* its position at the end of a swing of the water from A to B, then when the water was going back from B to A, in the next swing, the grain would be swept to *c* only, so that the net result of one complete oscillation would be the removal of the grain from *a* to *c*, *i.e.*, *towards the middle.*

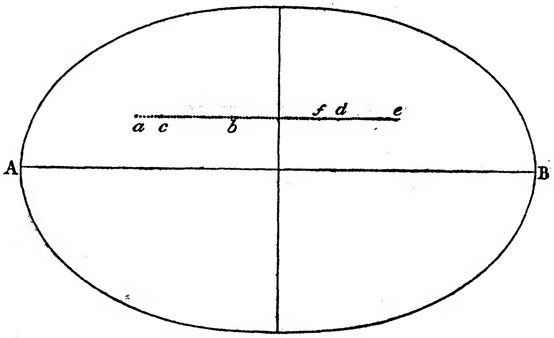

FIG. 2.—Change of Position of Sand Grains due to one Complete Oscillation of the Water.

On the other side, during the same complete oscillation, a grain at *d* would have been swept by the first swing to *e*, and by the second, past *d* to *f*, so that this grain also would have been moved *towards the middle.* Finally, therefore one complete oscillation would have altered the distance between the grains from *a d* to *c f*. In this way all the grains very soon reached the middle line

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* I use the word "ridge" to define an elevation, and a "ripple" to mean a ridge and furrow combined.

† I call each travel of the impulse from one end of the vessel to the other a "swing," so that there are two swings to each complete oscillation.

of the dish, and remained oscillating over it as long as the motion of the water was kept up.

It is clear that the formation first of the middle ripples, and the collecting of a mound across the middle of the vessel after prolonged oscillation could only be extensions of the operation just described. This, then, is the way in which the first ridge can form without the aid of any chance excrescence to start it.

When water is kept oscillating over sand, then, the dance of the grains described by Prof. Darwin is not, as he conjectured, a simple swaying to and fro, which would leave each grain where it found it, but for chance inequalities of the surface. *It is,* on the contrary, *a steady periodic advance from places where the horizontal velocity of the water is least to places where it is greatest, each oscillation leaving the grains nearer to their goal than it found them.*

Since water which rises and falls alternately at each end of a vessel, while its level remains nearly constant at the middle, is really oscillating in a stationary wave, of which the middle of the vessel is a loop, as regards horizontal motion, and the ends are nodes, it is interesting to note that the sand, in gathering across the middle of a vessel, is really collecting at a loop of the stationary wave generated by the oscillation of the water.*

2. *The Formation of Fresh Ripples beside an Existing One.*—Having established the primary ridge, and found the conditions necessary for its formation from smooth and level sand, the next question that arose was, how are all the other ripples started? Do they depend for their initiation simply on unevennesses of the surface, or are they also subject to some definite law? M. Forel† noted that a foreign body in the sand set up, in some way or other, a series of ripples in the sand on either side of it; but how these ripples start—whether all at one instant or each separately—and what is the process of initiation, has not, I think, hitherto been elucidated.

The solution of the problem cost me several weeks of observation and experiment, yet it was absurdly simple when it came. It was that a single ripple, existing alone, in otherwise smooth sand, initiates a ripple on

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* The terms "loop" and "node" have a perfectly definite meaning when applied to a vibrating string, for example, and mean the places of maximum and minimum motion. But when applied to a fluid oscillating with stationary waves, the terms are apt to be misleading, since in the place where the vertical motion is practically zero the horizontal motion is a maximum, and where the horizontal motion is practically zero the vertical motion is a maximum. Hence the node for vertical motion is the loop for horizontal motion, and *vice versâ.*

† 'Archives des Sciences Physiques et Naturelles,' Période 3, vol. 10, p. 39.

either side of it, that each of these ripples produces another on its farther side—these in their turn originate other ripples on their farther sides, and so on, till the whole sand is ripple-marked. This suggestion having occurred to me, I tried in many ways to make sure that it was correct. For instance, I formed a fairly high ridge at some distance from the middle of the vessel, and watched to see if others followed from that, before the primary ridge in the middle became visible; or, again, I made a ridge of some peculiar shape, such as this in plan, >, and noted whether the succeeding ridges took the same, or nearly the same, shape; and they did, the angle in each one being more obtuse than in that formed before it. Thus I felt sure of the *fact*; it only remained to see how it was accomplished.

For this purpose I abandoned my brown sand in favour of silver sand, which I had found to be so mobile that it spun in delicate fairy-like vortices in the lees of some of the ridges. These vortices, which differed widely from those discovered and described by Prof. Darwin, were, as I afterwards found, the true ripple-forming vortices, and I therefore call them ripple vortices.

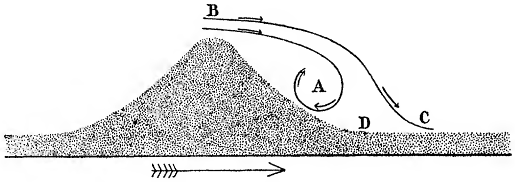

Fig. 3.—A Ripple-forming Vortex with Generating Ridge.

3. *Structure and Functions of the Ripple Vortex.*—The vortices I saw had horizontal axes and were spiral in shape, and they seemed to scoop sand out from the bases of the ridges, and to push some of it up the ridges while they carried the remainder whirling round with them. In watching them it occurred to me that since each vortex (A, fig. 3) raised sand up the ridge against which it revolved, while the water that flowed over the vortex BC (fig. 3) swept sand *away* from that ridge, there must be some neutral line parallel to the ridge at about D (fig. 3), on one side of which sand was being swept in one direction and on the other side in the other; and that, if that were so, a hollow must be formed parallel to the ridge, and reaching to some little distance on either side of the neutral line. Such a hollow must, it seemed to me, form a new furrow, while the wall of it on the side remote from the ridge must ultimately become a new ridge.

In order to see if my surmise was correct, it was necessary to simplify and magnify the action of the water, and also to see its stream lines. To simplify it I used only enough sand to cover the bottom of my glass trough ($36'' \times 6'' \times 8''$), and I got rid of sand on the ridge by making an artificial ridge of two china tiles sewn up in calico. To magnify the action, this ridge was made 2 inches high, and, so that no water should get round it or under it, it fitted tightly across the trough, and was jammed down on to the bottom.

Prof. Darwin employed ink squirted on to the sand to show him the stream lines of the water; but, as I found that this was impracticable with the violent motion necessary to raise sand vortices, I cast about to find an insoluble powder that had only a slightly greater specific gravity than water, so that it should move with the water in all its twists and turns. I found that well-soaked ground black pepper, after all the finest particles had been washed away, answered my purpose perfectly. With this I was soon able to follow the exact course of the water under every kind of condition, and fig. 4 shows what happened during a swing from left to right when the artificial ridge was in position, and when about a dessertspoonful of pepper was in the water.

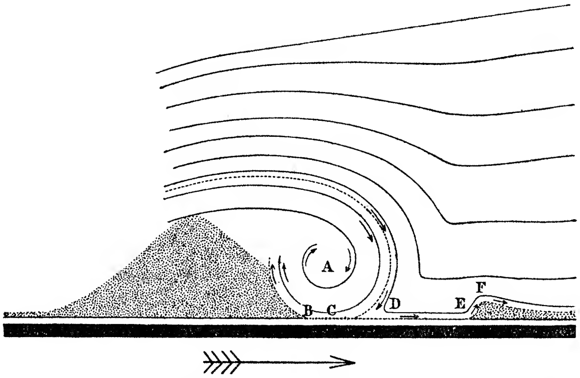

Fig. 4.—Ridge with Ripple Vortex, A; Neutral Line, D; and Brush, DEF.

Soon after the water began to flow over the ridge a small vortex appeared on its right-hand side. As water continued to pour over the ridge the vortex enlarged, so that its centre moved farther away from the ridge; finally there was a large vortex as at A (fig. 4), with water flowing down outside it, some of which, when it reached the bottom, struck the sand there *towards* the ridge, as at B, while the outer portion swept sand *away* from the ridge, as at D and E, leaving a bare space, which was what I have called the neutral line, between C and E, with its centre at D. With the return swing in the opposite direction, the vortex, with its whirling sand, was carried bodily over to some 3 inches on the other side of the ridge, the left side, where it dropped all the sand that had not fallen by the way. Then a similar vortex, a clear space, and a thickening of the sand to the left of it, appeared on the left-hand side of the ridge. These clear spaces were, of course, new furrows, and the sand walls farthest from the ridge were new ridges. After a few more oscillations not only did these grow into very decided ridges, but each of them had, in its turn, originated fresh furrows and ridges, and soon the whole of the space between the original ridge and both ends of the trough was covered with ripple-mark.

Since the water that strikes obliquely into the sand (DEF, fig. 4), after flowing over the vortex, clears a new furrow and sweeps up a new ridge exactly as a brush clears the floor and sweeps the dust up into a ridge, I shall call this water the "brush." The pepper, then, showed that the vortices and brushes cleared out new furrows and swept up fresh ridges exactly as I had imagined they did.

There are thus three ways in which sand ripples are originated, viz., the "uneven surface" method, the "differential motion" method, and the "brush" method. The first, which was the one described by Prof. Darwin, could, unaided, give rise to irregular ripple-mark only, since it consists in piling up sand in places where the surface chances to be uneven. The second is the method by which single ridges arise at the loops of stationary waves. The third is that by which the brush of the vortex in the lee of any existing dune or obstacle sweeps up a new ridge beside it, leaving a hollow along the neutral line of the vortex. The last two methods are evidently capable, unaided, of originating and then extending *regular* ripple-mark wherever water is oscillating over sand, whether its surface be smooth or uneven to start with. These last two, therefore, are clearly the *essential* methods, while accidental unevennesses of surface merely delay uniformity in the results.

4. *The Origin of the Ripple Vortex.*—Since the ripple vortex with its neutral line and brush plays such a large part in the initiation as well as in the growth of ripple-mark, it is clear that the causes of its formation and the laws of its development must be traced before any satisfactory theory of ripple-mark can be evolved. It is, as is shown in fig. 4, a spiral vortex composed of a ribbon of water wound round a cylindrical core, just as the mainspring of a watch is wound round its barrel. And one very curious fact that I observed about it was that it did not come into play in the beginning, but only during the latter part of each swing.

I had some difficulty at first in imagining how such vortices could arise, and sought in vain among all the vast number of papers on vortex motion that have been published within the last thirty years for some light on the subject. These appeared mostly to bear on vortices already formed, and moving in a frictionless fluid of infinite extent. Few dealt with any real fluid, and none, that I could find, with the process of formation of the vortex. Prof. Darwin traced analogies between the ripple-forming vortices and those set up by an oar in sculling, and by the motion of a fish's tail; but nowhere could I find any suggestion as to the actual process by which the vortices were generated.

Finally, I formed the hypothesis offered below, which is, I consider, confirmed by the experiments on the early stages of ripple-vortex formation presently to be described. When the impulse is in the direction AB, say (fig. 5), the water in the lee of a ridge, DE, is more or less protected from the impulse by the ridge, and therefore the water below D moves much more slowly than that above.

The first jerk of the impulse moves the water away from DE to EF, say, and then friction between the quick and the slow water enables the former to continue to drag the latter away from the ridge, and thus to keep up the diminution of pressure between DE and EF. The balance of pressure is, of course, immediately restored by a flowing in of water from other parts. *This diminution of pressure in the lee of a ridge is one of the two necessary conditions for creating a ripple vortex there.*

To find the other condition we must consider where the water comes from that flows in between the ridge and the water in its lee, when the pressure there is diminished.

In every mass of water oscillating about a single place of constant level, each swing distinctly divides itself into two parts—the first part (*a*, fig. 5), in which the higher half of the water falls, and the lower half rises to the average level; and the second part (*b*, fig. 5), in which that which was the higher half falls *below*, while that which was the lower half rises *above* the mean level. Thus, during every instant except the single one in which all the water is at the same level, the pressure exerted by every element of the water is compounded of two—one due to the velocity it has already acquired, and the other to the difference of level of the water at the moment. The second, which may be called "gravity pressure," is always, of course, exerted from the side on which the water is highest towards that at which it is lowest; and when there is a sudden attempt at a diminution of pressure at any one particular point, the direction of the gravity pressure at that point along each special line must be from the plane of highest level to the point. The gravity pressure at D, along the line PQ, for instance, when the pressure is diminished at D, must act from P towards D, and not from Q towards D, and similarly all the arrows in *a* and *b* (fig. 5) indicate the directions of the gravity pressures at D, along their own particular lines. In other words, when the pressure is suddenly diminished at any point in the water, the direction of the resultant gravity pressure at that point is from the end where the water is highest towards the point.

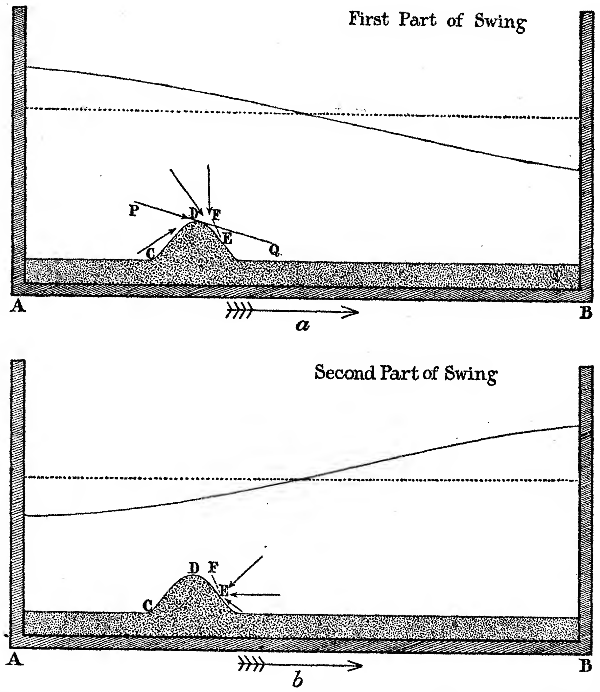

Fig. 5.—Diminution of Pressure between Ridge and Water and Direction of Gravity Pressure during Swing from Left to Right.

The effect of the gravity pressure on the flow of the water, when the pressure in the lee of a ridge is diminished by the drag exerted by the flowing water on the still water, can now be readily traced with the aid of fig. 5. The ridge is behind, and the water is being pulled away from it in front, so that water to fill the space can only flow from above or below. In the first part of the swing (*a*, fig. 5) the water below D can clearly exert no pressure along DE, and therefore it is only that above D which counts. This has a resultant gravity pressure along DE in a downward direction, which must impart at each instant an additional downward velocity to that which the water already possessed. Thus all the forces acting on the water in the first part of a swing tend to move it roughly in the same direction, and so no vortex then forms—the moving water simply sweeps over the ridge, pressing as closely as it can get against its lee side DE.

In the second part of the swing, when the water is highest on the *lee* side of the ridge, matters are very different—now the gravity pressure *opposes* the flow of the water, and its resultant along DE evidently points up the ridge (*b*, fig. 5), so that with a diminution of pressure at D the water close to the ridge flows in the opposite direction to that at a little distance from it, and so a vortex is created. *Thus the second condition for the formation of a ripple vortex is that the resultant gravity pressure along the ridge on its lee side shall tend upwards.*

As a test of the correctness of the explanation suggested above I studied some ripple vortices in water that was very shallow, and that therefore moved extremely slowly. As, under these conditions, pepper did not indicate the stream lines of the water with sufficient delicacy, I had recourse to crystals of permanganate of potash, each of which sent forth a steady stream of colour for several minutes, which indicated the flow lines of the water with great fidelity.

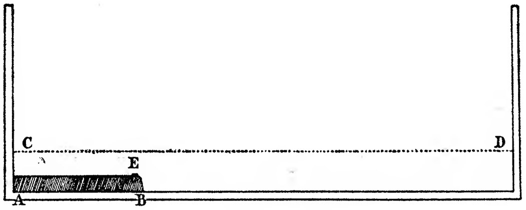

Fig. 6.—Arrangement for Watching Formation of Ripple Vortex.

To ensure a slow motion of the water I had it less than an inch deep, and, in order that my eye should not be distracted by vortices in both directions, I arranged the trough as seen in elevation in fig. 6. AB was a barrier which extended transversely across the trough but ended abruptly in a steep descent nearly parallel to the end of the trough. CD was the level of the water when at rest. At E a crystal of permanganate of potash was placed, protruding very slightly from the barrier. With this arrangement a good vortex formed when the swing was from left to right, but none, naturally, when it was from right to left.

The following was the process of formation in the left to right swing. When the swing began, a thin straight stream of coloured water flowed over the ridge, obviously filling in the gap made by the moving away of the water that was nearest the side of the ridge (*a*, fig. 7, Plate 3).* Next, when the water began to rise on the right-hand side, the flow over the top of the ridge became nearly horizontal (*b*, fig. 7), and, as the water beyond the

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* The end of the barrier and the water near it only are shown in the photograph.

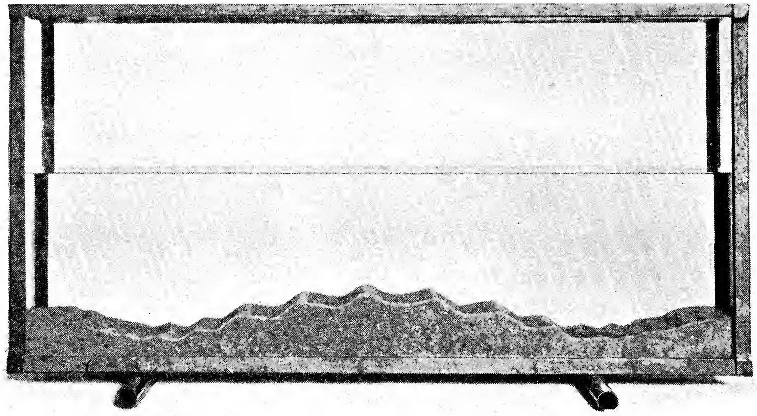

Fig. 1.—Vertical Section of Mound raised from Level Sand by Oscillating Water.

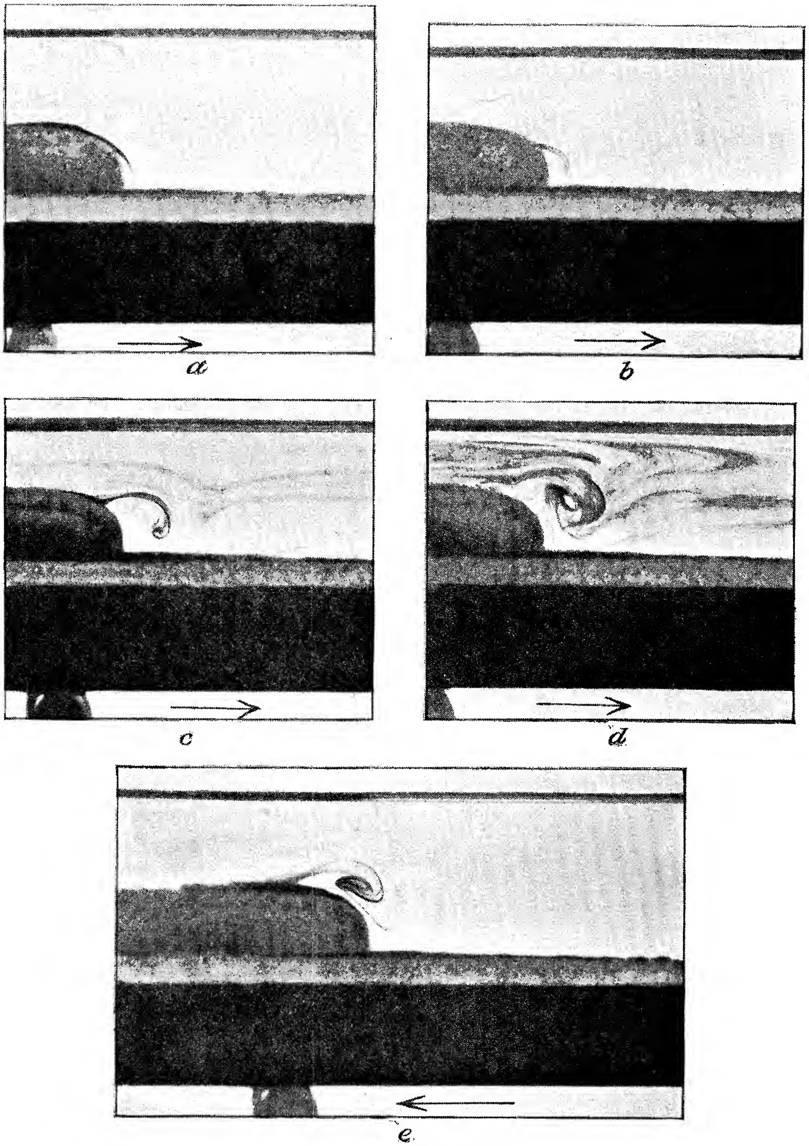

Fig. 7.—Different Stages of Formation of the Ripple Vortex.

ridge rose above the mean level, the downward-moving coloured stream *retreated upwards* as if being pushed up—as it undoubtedly was by the pressure of the higher water in front of it—the end of the horizontal branch curled back towards the ridge, forming a spiral, and the vortex was complete (*b*, *c*, *d*, fig. 7). Thus my hypothesis was confirmed in every particular. (In *e* (fig. 7), the direction of flow having changed, the vortex is being raised and tossed back along the barrier.)

There remained one further experiment to try in order to obviate all possible doubt. If, as I had surmised, and as the above experiments seemed to prove, it was necessary for the gravity pressure to *oppose* the flow of the water for the vortices to form, then no vortices of the kind I have described could be created by water flowing *steadily* in one direction over an obstacle, for in that case gravity pressure would always *aid* the flow of the water.

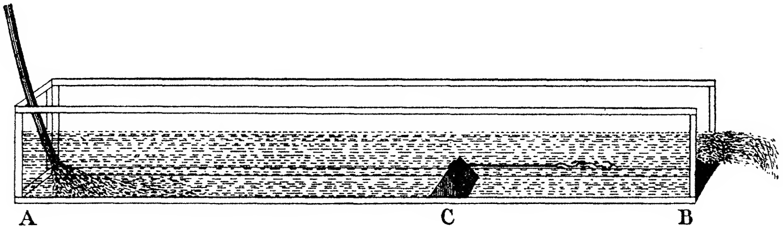

Fig. 8.—Coloured Stream from Crystal of Permanganate of Potash on Summit of Ridge over which Water is flowing steadily in one direction.

In making this experiment I used a trough, A B (fig. 8), open at one end, B, so that, if necessary, the water could flow out as fast as it flowed in. Water was led through an indiarubber tube into the closed end, and flowed out at the open end. A ridge of sand, C, extended across the trough nearer the open than the closed end, a good distance being left between it and the tube, so that the flow should become as steady as possible before the water passed over the ridge.

Closing the open end up to about twice as high as the ridge, so that still water rested against the ridge, the stream of colour from the crystal placed on the summit of the ridge flowed exactly horizontally for distances varying between 1 and 2 inches, according to the steadiness of the water, and then finished in a curious flickering motion, which is shown as nearly as possible in fig. 8*. Lowering the end barrier made no material difference in the appearance of the coloured stream, but when it was completely removed the

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* It is impossible to give an exact representation of this motion, because tiny water ripples came and went and flickered almost like the flame in a coal fire, which is now in one place, now in another.

rush of water flattened the sand ridge very considerably, still leaving the lee side very steep, however, and then the surface of the water was permanently lowered, and it rippled just beyond the ridge, but no ripple vortices were formed.

Finding that no water streamed down the ridge when the flow was steady, I concluded that the friction between the flowing and the comparatively still water was not sufficient to move the latter under steady conditions, but that a jerk was required to overcome the inertia of the stiller water. To try if this were so I gave the flowing water an impulse by pushing it along towards the ridge with a slip of wood the width of the trough. The result was that a small stream of colour immediately appeared against the side of the ridge, and, as soon as the water in front of the ridge rose higher than that behind, a vortex formed.

These experiments appear to me to prove conclusively that the creation of ripple-forming vortices depends entirely upon the two conditions I have formulated, viz.: (1) the establishment of a diminution of pressure in the lee of an obstacle; and (2) the turning down and back again of the water that has flowed over the ridge, through the backward pressure on it of the water that is temporarily raised above the main level in the far end of the vessel.* Thus ripple vortices depend for their existence on variations in the gravity pressure of oscillating water that obviously must exist, but the importance of which in connection with the formation of vortices has not, perhaps, before been recognised.

It has hitherto been supposed that a uniform steady current of sufficient velocity was capable, unassisted, of generating vortices, both of the sand-raising type and of other types, when flowing over an uneven sandy bottom. My experiments given above, and others that I hope to publish later, show, however, that although the water in the lee of an obstacle over which there is a steady flow is disturbed, *no definite vortex in the least resembling a ripple vortex forms there. Hence a steady current is unable either to generate or to maintain ripple-mark.*

5. *Different Intensities of the Ripple Vortices.*—The ripple vortices have not all the same intensities, and at the ends of the vessel they even differ so widely on the two sides of the same ridge that the vortex facing the middle of the trough in one swing is incapable of moving the sand, while the end-facing one in the next swing raises a pretty little sand vortex. A little consideration shows that this must be so.

Ripple vortices are caused by a current flowing over the summit of a ridge

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* I am aware that this explanation has been contested, and I offer it entirely on my own responsibility.

away from its lee, acted on by a pressure exerted *towards* the lee. The rotational velocity of the vortex must depend upon the horizontal speed of the current and the magnitude of the pressure combined.

Now the horizontal velocity of the current increases from zero at the near end of the vessel (the end at which the impulse is given) to a maximum at the middle, and diminishes to zero again at the far end. Hence, if the nine dots in fig. 9 represent the summits of ridges during the second part of a swing from left to right (remembering that no vortices form in the first part of the swing), the upper arrows will represent the direction, and their lengths, roughly, the relative horizontal velocities of the current. Again, since the greatest gravity pressure is always at the far end of the vessel, in the second part of the swing, this pressure is least at the near end of the trough, and increases continuously to the far end, so that the lower arrows in fig. 9 give the direction of the motion imparted by the gravity pressures, and their lengths represent roughly the relative values of those pressures. Thus, both current and pressure are small at the near end of the vessel, but, while both increase up to the middle point, beyond that the current diminishes and the pressure continues to increase. Hence, in a swing from left to right, the rotational velocities of the vortices must be roughly proportional to the sizes of the vortices in fig. 9—that is they increase from the near end up to the middle ridge, and then diminish again to the far end, but each is larger on the far side than the corresponding one on the near side of the vessel. In the next swing—from right to left—the relative velocities of the vortices on the two sides of the vessel would, of course, be changed, so that the smallest vortex was now on the right-hand side of the ridge. It is clear, therefore, that while the rotational velocity of the end-facing vortex of a ridge is always greater than that of its middle-facing vortex, both are swifter the nearer the ridge is to the middle of the vessel.

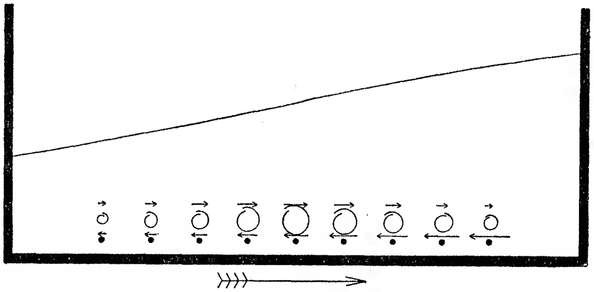

Fig. 9.—Rotational Velocities of Vortices at the same Instant in Different Parts of Trough during Swing from Left to Right.

6. *Initial Ripple-distance.*—Prof. Darwin observed that the ripple-distance increased with the number of oscillations of the water after its initiation, but, in the absence of any information as to the process of initiation, it has been impossible for any one hitherto to judge of what determines the first distance of one ripple from another. We can now, however, ascertain this from a consideration of the laws governing the lee vortex and its brush.

In smooth and level sand with a single ridge in it, the first swing of the water causes a vortex to form in the lee of this ridge, the brush of which sweeps together a line of sand that is the nucleus of a new ridge (fig. 4, p. 290). Clearly the distance between this new ridge and the old one is determined by the final size of the vortex in that first swing and the distance between its neutral line, D (fig. 4), and the new ridge, EF, which distance I shall call the sweep of the brush. As this latter is the more important factor of the two in determining the ripple-distance I shall deal with it first.

The Sweep of the Brush.—The sweep of the brush depends upon two things, viz., the velocity of the water that forms it and the room it has to spread in. From the nature of the brush its velocity must vary with the horizontal velocity of the water passing over the ridge, and this is greatest in the middle of the vessel, greater the greater the amplitude of the wave, and, since the horizontal velocity varies less, in different parts of the vessel, the deeper the water, the sweep of the brush also varies less, in different parts, the deeper the water.

The room the brush has to spread in evidently depends, not only upon the length of the trough and on where the generating ridge is in the trough, but also upon whether the vortex is on the end-facing or the middle-facing side of the ridge. Thus the sweep of the brush is greater—

(1) the longer the trough, *i.e.* the greater the length of the stationary wave,

(2) the farther the lee side of the generating ridge is from the end that it faces of the vessel,

(3) the greater the amplitude of the wave.

(4) It differs less, in different parts of the vessel, the deeper the water.

The Size of the Vortex.—As the vortex is a band of water wound round a cylinder of originally stiller water, its size at any given instant is greater the larger the cylinder is to start with, the more quickly it is wound and the longer it spins.

Now the thickness of the cylinder to start with depends on the height of the generating ridge, within certain limits to be determined later. The velocity with which it is turned has been shown to depend on the horizontal velocity of the water that rubs the sand, and on the backward gravity pressure, both of which are functions of the amplitude of the wave and of the position of the vortex in the trough and on the ridge. The duration of the vortex depends upon the periodic time of oscillation, which bears a direct relation to the length of the trough and is an inverse function of the depth of the water.

The vortex is larger, at any instant, therefore—

(1) the higher the generating ridge (within limits),
(2) the greater the amplitude of the wave,
(3) the nearer the generating ridge is to the middle of the trough.
(4) It is greater on the end-facing than on the middle-facing side of the generating ridge.
(5) It is greater the longer the trough and
(6) the shallower the water (within limits).

The conditions necessary for the vortex to be large, then, are very similar to those required for the brush to have a long sweep, except in one particular—the vortex is larger on the end-facing side of the ridge, and the sweep longer on its middle-facing side. When two conditions are at war, experiment must prove which gets the upper hand. I used my artificial ridge to try this, and found that the new ridge created on its middle-facing side was always farther from it than the one created on its end-facing side, but that the two distances differed less the nearer the artificial ridge was to the middle of the vessel.

We can now, by combining the conditions necessary for the vortex to be large with those requisite for the brush to have a long sweep, find out how the initial ripple-distance varies with varying conditions. This distance must increase with—

(1) the height of the generating ridge (within limits),
(2) the amplitude of the wave,
(3) the nearness of the generating ridge to the middle of the trough,
(4) the length of the trough, or of the stationary wave,
(5) the shallowness of the water (within limits).
(6) It is greater on the middle than on the end-facing side of the generating ridge.
(7) It differs less in different parts of the vessel the deeper the water.

The two limits mentioned above are the following:—

(1) If the first ridge is so high that the velocity of the water is only sufficient to move a small portion of the slack water in its lee, the diameter

of the cylindrical core is determined by the velocity of the water instead of by the height of the ridge. In that case the new ridge rises *on* the first one instead of beyond it.

(2) When the water is so shallow that its friction is insufficient to move the particles of sand, no ripples form.

7. *The Growth of Ripple-mark.*—Let BCD (1, fig. 10) be a ripple and AB, DE, the sides nearest to it of the two neighbouring ripples, when the water was at rest; what is seen when it is oscillating is this:—

In a swing, from left to right, say, at the first instant no vortex forms on either side of BCD, sand is simply pushed up BC, and a little is washed off the summit C, and thrown or dropped anywhere between C and E (2, fig. 10). Next a vortex forms against CD, which is made apparent, if the motion is strong enough, even without the use of pepper, by the whirling round of the sand scooped out by it from the lower parts of CD (3, fig. 10), and by the movement of the sand up DC against the general flow of the water. At the same time sand continues to be pushed up BC to the summit, where a part of it rests, while the remainder slides over to the other side, and, being kept from falling by the vortex, gathers into a small ridge such as F (3, fig. 10). Then when the swing changes, the sand in the vortex rises, spreading out as it goes (4, fig. 10). The part of this sand that touches the summit C remains there, and the rest is carried along to the left, to a distance varying between a fraction of an inch and several inches, according to the horizontal velocity of the water and the periodic time of the oscillation. Wherever this vortex has reached when the swing changes, all the sand that has not fallen by the way is deposited, unless the motion is so violent that some is carried back again in the next swing, before it drops. The first flow of the water in the new swing also sweeps the ridge F up to the summit and leaves it there (4, fig. 10), so that this helps to raise the summit. After this a vortex forms against BC (5, fig. 10) as it did previously against CD, and at the end of the swing the ripple resembles BCD (5, fig. 10), B being lower and C higher than in 3 (fig. 10). Thus every swing raises the summit and lowers the base of the ripple and so increases its height doubly. When the oscillations are allowed to die away, the small ridges at F and G raised by successive swings are smaller and higher up with each oscillation, till, when the water comes to rest, the ripple resembles 6 (fig. 10), C being higher and B lower than when the oscillations started.

All this is what is *seen* when ripples are growing under oscillating water. The explanation of most of it is obvious, from what has gone before. The first instant of observation, during which no vortex forms, is what I have called the first part of the swing—the time when the water is gaining its mean level, and there is, consequently, no backward gravity pressure. The next moment, when the sand vortex forms, is what I have called the second part of the swing, when the water is rising above its mean level, and there *is*, therefore, a backward gravity pressure. The only part of the operation that has not already been accounted for is the expansion and rise of the vortex when the swing changes. This I explain in the following way:—

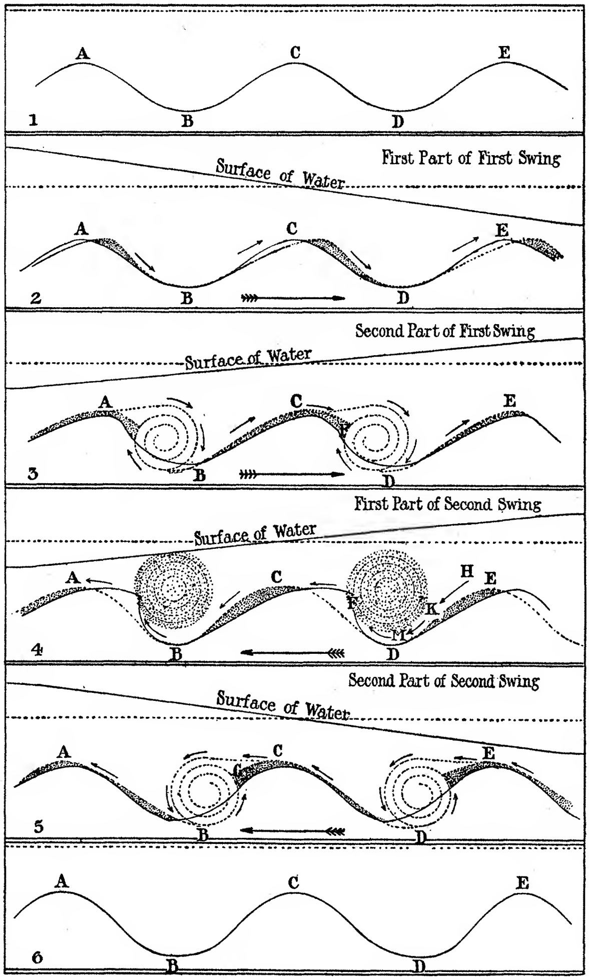

Fig. 10.—Various Changes in Size and Shape of Ripple produced by one Complete Oscillation of the Water.

The vortex is formed and grows under pressure from the onward flowing water above and the backward pressed water below. At the moment when the swing changes, the water as a whole comes to rest, with the result that the downward pressure of the flowing water is taken off the vortex, while the upward pressure from the raised water remains (HKM, 4, fig. 10). The moment the downward pressure is taken off, the vortex expands, I imagine owing to its centrifugal force, and is raised upward by the pressure of the high water, and it is then swept onward in the new direction with the new swing.

8. *The Motion of each Ripple as a Whole.*—Concerning the growth of the ripple-distance, Prof. Darwin says: "On the parts of the plate where the sand is thick, a continual rearrangement of ripple-mark goes on; the wave-length extends by the excision of short patches of intercalated ripple-mark, and by general rearrangement. Finally, the sand reaches an ultimate condition as regards wave-length, although rearrangement of ripple-mark still appears to go on for a long time."

My own earlier observations apparently accorded with this description, but, wishing to find out more definitely exactly how the rearrangement took place, I started *de novo* with smooth level sand, in the middle of which I had artificially made two ripples as starting points for other ripples. These were raised by pressing a thin slip of wood down into the sand and then withdrawing it, after moving it slightly to each side, to pile up the sand (*a*, fig. 11, Plate 4). The ripples were about half an inch high on the inner side to begin with. With this arrangement I was led to the following new results:—

After a few oscillations, each of the ridges (*b*, fig. 11) was half an inch or so farther from the middle of the vessel than at starting, while the two new ridges that one would have expected, from what has gone before, had formed to the left and right of them.

Further oscillations increased the spaces between all the ripples, as well as producing more ripples to the right and left of them (*c*, fig. 11). Thus, for a short time after its initiation, each ripple travels farther from the one that generated it, and, consequently, from the middle of the vessel, with every oscillation. After a little while—some time before the whole of the sand was rippled—the summits of the two ridges that were nearest the middle of the vessel first became stationary, and then began to move back *towards* the middle with each oscillation. Next the same thing happened to the next ridge on either side of the middle ones, and finally—soon after the whole of the sand had become ripple-marked—all the ripples were travelling *towards* the middle of the trough. Now it is obvious that if all the ripples travel towards the middle of the vessel, at least two of them must at last coalesce, and then two others, and so on; and this does actually happen. As soon as two ridges get so close together that the vortex of one can snatch sand from the summit of the other, while that other, from inferiority either of size

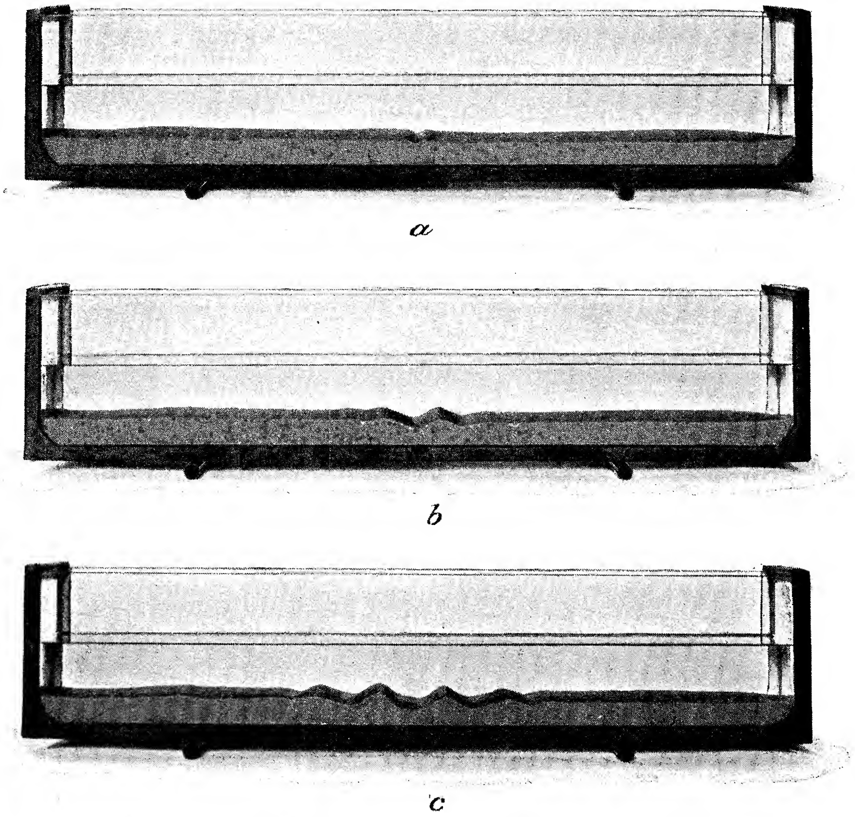

Fig. 11.—Motion of Ripples immediately after Initiation.

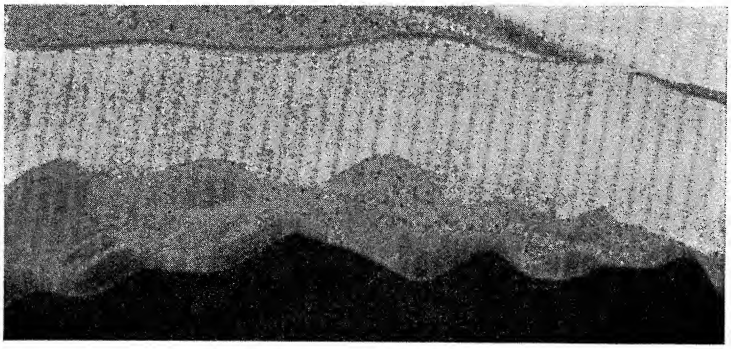

Fig. 13.—Longitudinal Scoring of Sand Ripples.

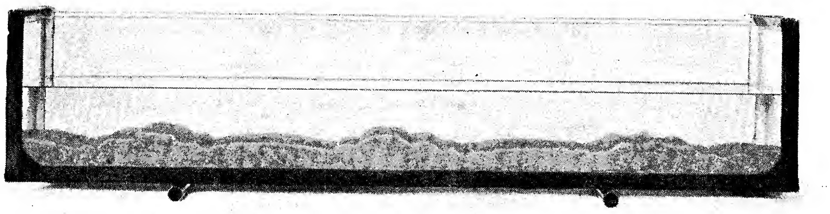

Fig. 14.—Sand Mounds formed by Water Oscillating in Stationary Waves two-thirds of the Length of the Trough.

or position, or both, is incapable of returning the compliment, this second one quickly gets swallowed up, and its sand goes to swell its rival. These coalitions generally take place at or near the middle of the trough, and they would naturally always do so but that the sand, water, and impulses are never perfectly symmetrical. For the same reason the whole length of a ripple seldom gets absorbed at the same time. It is usually higher in one part than in another, and then the lower part disappears first, while sometimes the higher joins on to the demolishing ripple that is on its end-facing side, as shown in plan in *a*, fig. 12, where AB is the part, and CD the whole ripple that it joins. When this happens, succeeding oscillations usually bring AB and BD into line, cutting off CB (*b*, fig. 12), which then joins the next ripple (*c*, fig. 12), when the same process is repeated, and EB is cut off and joins the next ripple, and so on till at last the cut off part either becomes absorbed or obliterated, or it grows into a full-sized ripple at the end of the trough.

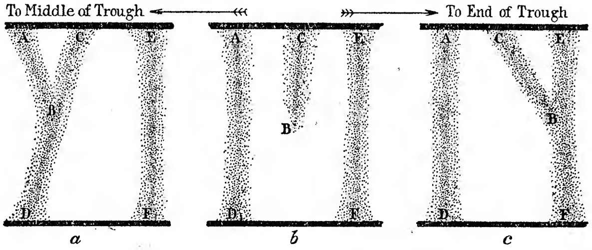

FIG. 12.—Formation and De-formation of Y-shaped Ripples. Plan.

For some little time after the whole sand has become rippled, in a trough, the ripple-distance at the ends is only slightly less than at the middle; but as the height of the middle part of the sand grows, while that of the ends diminishes, under continued oscillation, the ripple-distance increases in the middle while it diminishes at the ends, so that at last the series of ripple-distances is graduated from very small—perhaps half an inch or so at the ends of the vessel to three or even four inches in the middle. All these changes of ripple distance are entirely effected by the coalition of ripples just described, together with the formation of new ripples at the ends of the trough, and these two phenomena are due to the *motion* of the ripples as wholes, which is brought about in the following manner:—

It is easy to see that the motion must be connected with the transference of sand from one side of a ridge to the other and back again during each oscillation (fig. 10, p. 301). If more sand were always deposited on the right side in one swing than was swept back on to the left side in the next, for instance, the middle plane of the ridge would travel from left to right, and the whole ridge would therefore do likewise; and, *vice versâ*, an excess of sand placed on the left side would lead to the motion of the ridge from right to left. I shall now show that this unequal transference of sand is exactly what *must* take place among a series of ripples.

It has been seen that during the second part of a swing (3, fig. 10) a vortex scoops up sand FD from the lower part of one side of a ridge, and that during the first part of the next swing it carries this sand over to the other side of the ridge, together with a slice of sand which it cuts off from the upper part of the first side (4 and 5, fig. 10). The amount of sand scooped out by the vortex is greater the larger and swifter the vortex; also the slice of ridge carried over the summit with the vortex sand increases with the depth of the side of the ridge and with the longitudinal velocity of the water, and is greater when this velocity is increasing than when it is diminishing.

Now, I have shown that in established ripple-mark, the end-facing vortex of a ridge is always larger and swifter than its middle-facing vortex. Also, the flowing water has always an increasing longitudinal velocity when it is washing sand from the end-side to the middle-side of a ridge, since it is then travelling *towards* the middle of the vessel, and a diminishing velocity when it is carrying the sand in the opposite direction, *away* from the middle. It follows, therefore, that in established ripple-mark, more sand must be transferred from the end-side to the middle-side of a ridge in one oscillation than is conveyed back again to the end-side in the next oscillation, and it is for this reason that the ridge, as a whole, travels towards the middle of the vessel. In fact, *each established ripple partakes, as a whole, of the general movement of the sand towards the places where the longitudinal velocity of the water is a maximum.* It is thus apparent that, unless there is little enough sand for it all to collect at the middle, the ripples must continually re-arrange themselves, not only for a time, but for as long as the oscillation is continued, or till the sand is piled up in such a way that ripples are no longer possible.

The apparently anomalous cases when the ripple moved away from the middle are readily explained. These were: (1) when the first two ripples were made by pressing a slip of wood down into the sand in the middle of the vessel, so that the furrow between the ridges was deeper than the furrows on their other sides (*a*, fig. 11); and (2) when a new ripple was generated by the end-facing vortex of an existing one (*c*, fig. 11, Plate 4). Both these ripples had deeper middle-side than end-side furrows—the first, because it was specially made in that way, and the second, because the vortex on the middle-facing side of the ridge had scooped out the furrow below the mean level of the sand, whereas no vortex had yet existed on the end-facing side, which was merely a small pile swept up above the level of the sand (*b*, fig. 11). Hence, at first, the end-facing vortex of each ripple was smaller than its middle-facing vortex and the end-side of the ridge was shallower than its middle-side, for both of which reasons less sand was at first scraped off the end-side of the ridge in one swing than was returned to it, from the middle-side, in the next; and, consequently, the ridges travelled endwards. As, however, the end-facing vortex has always a greater rotational velocity than the middle-facing vortex (see p. 297), the end-side furrow deepened more quickly than the middle-side one. Hence, the disparity in the quantity of sand that was scraped off each side alternately gradually disappeared, and when this became equal the ridge became stationary. As the end-side furrow still continued to deepen more rapidly than the middle-side one, however, *more* sand was now scraped off the end-side than was returned to it, and so the normal state of things was brought about, viz., the ridge travelled as a whole towards the middle of the vessel.

In order to test the validity of the foregoing argument, I made two ripples in the middle of the vessel by scraping up sand from both ends, and then pressing down the slip of wood into the middle of the resulting dune, so that the end-sides of the two ripples thus formed were longer than their middle-sides. The result was as I expected. After a few oscillations, instead of the two dunes having separated further, they had coalesced and become one, through each having moved towards the middle.

Other experiments of the same kind, such as making a ripple about half-way between the end and the middle, and first having its deeper furrow on the end-side, and then on the middle-side, were equally successful; for, in the first case, the ripple moved towards the middle, and in the second, towards the end of the vessel.

9. *Side of Ripple facing Shore Steeper than Side facing Sea.*—The unequal sizes of the vortices on the two sides of a ridge and the unequal transference of sand from one side to the other naturally affect the shape of the ripple. The larger end-side vortex hollows out a deeper end-side furrow, tending to make the end-side of the ridge steeper and deeper than the middle-side, a tendency that is strengthened by the greater transference of sand to the middle-side, and that is still further accentuated by the sand dropped by the travelling vortices, which naturally rests more easily on the gentle slopes than on the steep ones (fig. 1, Plate 3). This is the reason that the ripples on the sea shore generally have steeper slopes facing the shore and gentler slopes facing the sea. The shore takes the place of the end of the vessel in checking

the flow of the water, so that the shore-sides are the end-sides of the ripples, and the sea-sides are what I have called their middle-sides.

10. *The Longitudinal Markings of Rippled Sand.*—Sand ripples are never entirely smooth, they are marked with small irregular longitudinal ridges and grooves, almost as if they had been combed (fig. 13), more especially when the motion of the water is fairly violent. This is due, I think, to the vortices not having the same cross-section throughout their whole length, but being rather like beads of varying diameters strung on the same horizontal wire, so that they scoop out the ridges to unequal depths, according to the cross-section of the beads. With unequal cross-section of the vortices, the brushes would be unequal also, and so they would dig to different depths into the weather sides of the ripples. Thus, both sides of the ripples would be scored. The following is the way in which I believe the vortices obtain their varying cross-sections:—

During their growth they have to sustain the weight of such ridges as those marked F (fig. 10, p. 301). But from accidents of slipping this ridge has very varying weights in different parts. Where the weight is heavy, the sand slips further, and the vortex is necessarily smaller than where it is light. Hence, each vortex is not single, but is composed of a series of vortices of different diameters set end to end.

When the swing changes, the series breaks up into a number of separate small vortices, each having its axis inclined to the horizontal at some angle that depends upon the contour of the sand against which it was twirling at the moment when the swing changed, and it was pushed back and up. The inclination of the axis to the horizontal is sometimes so great that it actually becomes vertical, and but few of the vortices remain horizontal after they have begun to travel.

11. *Travelling Vortices.*—We have now traced three steps in the advance of the sand from the ends to the middle of the vessel:—

(1) Before the ripples have formed, every complete oscillation of the water leaves each single grain that is moved nearer to the middle than it found it.

(2) While the ripples are forming, the same process goes on in the parts of the sand where ripples have not yet appeared; but, where they have, each ripple is removed *farther* from the middle with every oscillation.

(3) When the whole sand is ripple-marked, every oscillation leaves each whole ripple nearer to the middle than it found it.

There remains one more step to complete the series. When the vortices are carried away from their generating ridges, after each new swing, it is the large vortices from the *end*-facing sides of the ridges that are transported towards the middle of the trough, while the small ones from the *middle*-facing sides are taken in the opposite direction. The sand that is continually dropping from them, therefore, falls more thickly on the middle of the vessel than on its ends, so that in this way also the gathering of the sand in the middle is accelerated. Thus, every operation performed by the water—except one which is purely temporary—tends to drive the sand from the ends to the middle of a vessel, when it is oscillated in such a way that the water level remains constant in the middle, and has its maximum variations at the ends. No wonder, then, that the sand soon becomes banked up in the way shown in fig. 1.

12. *Ripple-Mark on the Sea-Shore.*—For the ripple-mark that I have hitherto considered, one of two conditions was necessary to start the ripples. Either the oscillating water had to have at least one constant place of maximum longitudinal velocity, or there must be at least one obstacle or hollow in the sand to give birth to vortices. Of obstacles in the shape of stones, bunches of seaweed, etc., as well as of hollows, there is generally no lack on a sandy shore, but the question arises—would a beach that was entirely smooth and featureless to begin with become ripple-marked when the sea oscillated over it? Obviously it would, for in order to start a primary ripple it would only be necessary for the water to have a maximum longitudinal velocity at or near the same place during a few successive oscillations, so that even a single one of the smallest of small primary ripples could form. Once this was done, other ripples, as I have shown, would arise as a matter of course, until the whole sand was covered with ripple-mark; and the ripple-mark would be very regular, for although the places of maximum horizontal motion of the water would often remain the same, or nearly the same, for a short time, yet they would not usually remain so long enough for such mounds as those shown in fig. 1 to form. On the contrary, the velocity and back gravity pressure of the water flowing over any individual ripple must be continually changing, so that on the average each is subjected to the same influences, and their heights and ripple-distances are therefore fairly equal after any single tide. But as the heights and ripple-distances of all vary with the amplitude of the wave, they are greater after a storm than during calm weather.

In order to try and imitate the action of the sea on an originally smooth sandy beach, I used about an inch of sand made quite smooth and level in my 36-inch trough, with about 2 inches of water above it. I then alternately raised and lowered one end of the trough, bit by bit, giving it a slight jerk at each rise or fall, to make the water oscillate, but allowing several oscillations to take place between each jerk. In this way not only did the place of maximum longitudinal motion differ slightly for each oscillation, but the depth of the water changed at the same time, which is exactly what happens in the sea. When this was done for some time, the sand remained level on the whole, but became covered with very regular even ripple-mark, just like that of the sea-shore; and, as was to be expected, the sand did not rise higher, on the whole, in any one place than in any other.

It might be objected that, since all the conditions for the formation of ripple-mark appear to exist in every place where there is a sandy beach washed daily by the tides, ripples ought to be much more universal in such places than they are. The reply is that they *are* universally *made,* but that most of them get smoothed out by the edges of the waves of the retreating tide, and only remain intact in pools or depressions of any kind, where the water, being temporarily left by the retreating tide, sinks slowly and uniformly downwards through the sand.

13. *Ripple-Mark under Stationary Waves.*—In my earlier experiments the trough was oscillated in the simplest possible way, *i.e.* so that the length of the stationary wave was twice the length of the trough. Subsequently it occurred to me that by oscillating more quickly I ought to be able to get smaller stationary waves, the effect of which on the sand would be to produce groups of ripples at or near the loops of the waves, where the longitudinal velocity was a maximum, and to leave smooth spaces at or near the nodes, where it was a minimum. In this I was successful, and fig. 14, Plate 4, shows the effect when the stationary wave was two-thirds of the length of the trough instead of being twice as long. Ripple-marked mounds rose at the three loops, where the level remained constant and the longitudinal velocity was a maximum, and smooth spaces remained at the four nodes. By oscillating the trough with different velocities I was able to obtain from one to seven mounds, with clear spaces between, at will.

The height to which a mound can rise when the water has a given minimum depth and the quantity of sand is practically unlimited depends solely on the length of the stationary wave. This wave-length settles the distance between the mounds, and this distance determines the height to which a mound will rise before some of it slips down, or before the weight of the highest part makes it spread out laterally, thus partially filling up the space between it and the next mound, and so diminishing its height again.

The following is what happens when oscillation is prolonged:—If the sand is thin, it collects in separate mounds at the places of maximum longitudinal velocity, leaving the bottom of the vessel bare between. If the sand is of practically unlimited thickness it rises to a certain height at each of such places, and then broadens on the top into a sort of tableland, smooth at first, but covered with irregular ripple-mark afterwards. When this happens, the mound has reached the maximum height it can attain to under the given conditions. Soon sand slips somewhere, and the whole shape changes somewhat, and then rebuilding goes on, till the same height is reached again, and then another slip occurs—sometimes two or three in different places—and rebuilding starts afresh. There is thus no *exact* height which the mound *retains*, but it has a *maximum* height for each wave-length, provided the water has a certain minimum depth above the top of the mound when it is at its highest. Increased amplitude of the wave hastens the attainment of this height, while greater depth retards it; but nothing but a change of wave-length *alters* it, I think.

When the oscillation is very regular, the bare spaces between the mounds remain entirely free from ripple-mark, as in fig. 14, Plate 4, but if it is at all irregular, there are no parts of the sand where the level of the water remains *quite* constant, and then a series of ripple-marked heaps rise which are separated by no smooth spaces.

In shallow water, after a time, the tops of the mounds become flat, and the furrows then look like grooves cut for irrigation purposes. In one case, for instance, the mounds finally became quite flat-topped, with runlets between, as shown in profile in fig. 15.

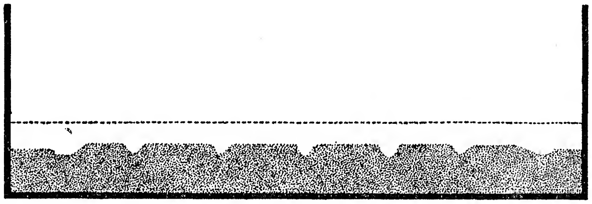

Fig. 15.—Flat-topped Mounds with Grooves between, produced by Prolonged Oscillation with Stationary Waves in Shallow Water.

The process described above, by which mounds of sand are built up under the action of stationary waves, may be summarised as follows:—

(1) Ripples always first form at places of maximum longitudinal motion of the water, *i.e.* at places of constant level.

(2) Smooth spaces are left at and near the places where the water has no longitudinal motion, that is, where the change of level is greatest.

(3) Each set of ripples grows into a mound, having its centre plane at the place of maximum longitudinal velocity and its lowest parts close to places where the longitudinal motion is zero.

(4) There are therefore two of these mounds to each water wave.

After having arrived at the above conclusions, and proved them experimentally, I found that C. de Candolle,* in a long and most interesting paper on the subject, had mentioned that, under stationary waves, he had observed groups of ripples with smooth spaces between. He said: "Lorsqu'il existe dans l'auge plusieurs ondes stationnaires, il se forme autant de systèmes de rides qu'il y a de ces ondes" (this would only be correct if C. de Candolle used the term "onde" to signify a half of what we call a stationary wave). He continued: "Et ces rides sont groupées de part et d'autre de *chaque plan vertical séparant deux ondes contiguës.*" [The italics are mine.] This appears to be vague, but the context shows that C. de Candolle meant the places of constant level, and it is, therefore, a true description. He appears, however, to have failed to notice the most important point about these groups of ripples, namely, that they finally grew into mounds, nor do I think that this fact has ever been previously mentioned.

With regard to other mounds formed by stationary waves, I should suggest that the tidal sand ridges first discovered by Prof. Osborne Reynolds† in his model estuary were thus produced, and that the similar ridges to be found in natural estuaries, many of which Dr. Vaughan Cornish has so ably described and beautifully photographed, have a like origin.

Besides tidal ridges, it seems probable that the chains of sand banks in the sea and of sand dunes on shore, as well as the gigantic sand heaps in the Asiatic deserts described by Sven Hedin,‡ originate and grow under the action of waves that are stationary for longer or shorter periods, while the smaller ripples that cover all of them are due to vortices similar to those that I have shown to produce the ripple-mark of the sea-shore.

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* "Rides Formées à la Surface du Sable Déposé au Fond de l'Eau," 'Archives des Sciences Physiques et Naturelles,' Période 3, vol. 9, p. 256 (1883).

† "First Report of Committee on the Action of Waves and Currents," 'Report of the British Association,' 1889.

‡ 'Central Asia and Tibet,' by Sven Hedin, vol. 1, pp. 248, 278.

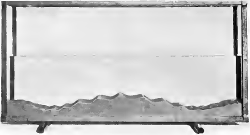


Fig. 1.—Vertical Section of Mound raised from Level Sand by Oscillating Water.

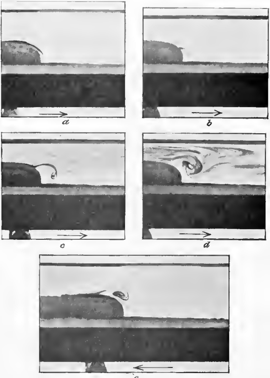
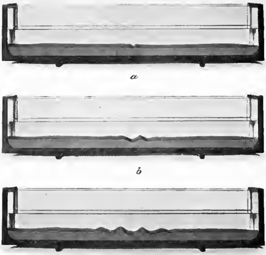


Fig. 7.—Different Stages of Formation of the Ripple Vortex.



a

b

c

Fig. 11.—Motion of Ripples immediately after Initiation.

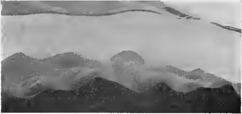


Fig. 13.—Longitudinal Scoring of Sand Ripples.

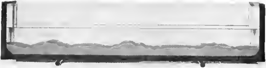


FIG. 14.—Sand Mounds formed by Water Oscillating in Stationary Waves two-thirds of the Length of the Trough.